Black hole to cosmic horizon microstates in string/M theory: timelike boundaries and internal averaging

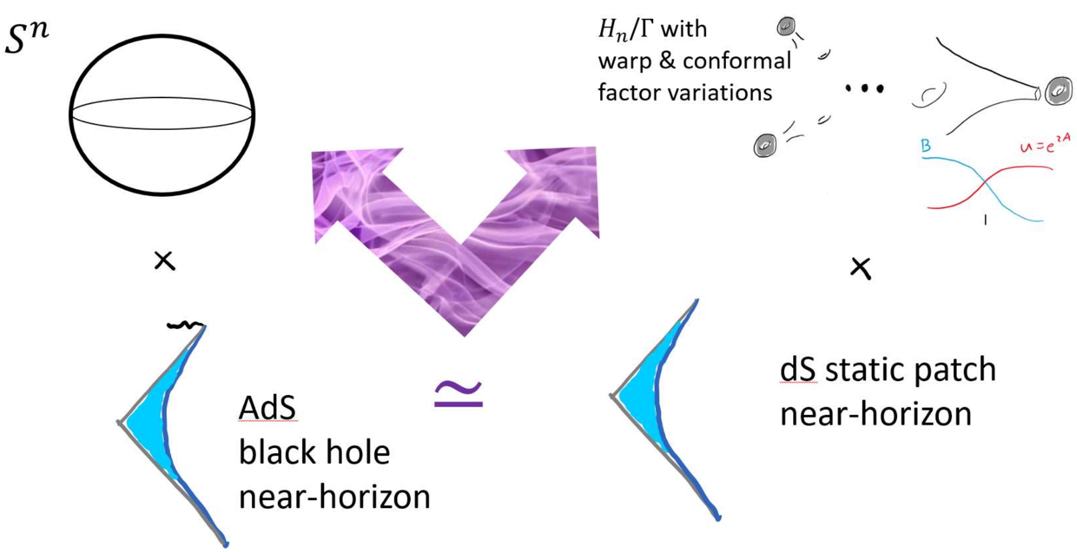
Based on E.S. 2212.00588 [hep-th],

+ earlier works --
de Sitter Microstates from $T\bar{T} + \Lambda_2$ and the Hawking-Page Transition
arXiv:2110.14670, *JHEP* 07 (2022) 140
w/ Evan Coleman, Edward A. Mazenc,
Vasudev Shyam, Ronak M Soni, Gonzalo Torroba, Sungyeon Yang

+ others w/ Alishahiha et al, ...,Dong, Gorbenko, Lewkowycz, Liu, Torroba...
+ works in progress w/Batra, De Luca, Shyam, Soni, Torroba, Yang,…

Outline: Solvable $T\bar{T} + \Lambda_2$ theory interpolates between AdS Black hole & dS static patch, bringing in the boundary to the BH horizon -- indistinguishable from any other horizon -- then moving boundary out in a way that reconstructs the dS static patch bounded by a Dirichlet wall. This matching is at large temperature, compelling a thermal mixture among all internal configurations consistent with horizon => extends to UV (M/string theory)

Also can't tell the difference between dS and AdS/BH horizons **internally**!

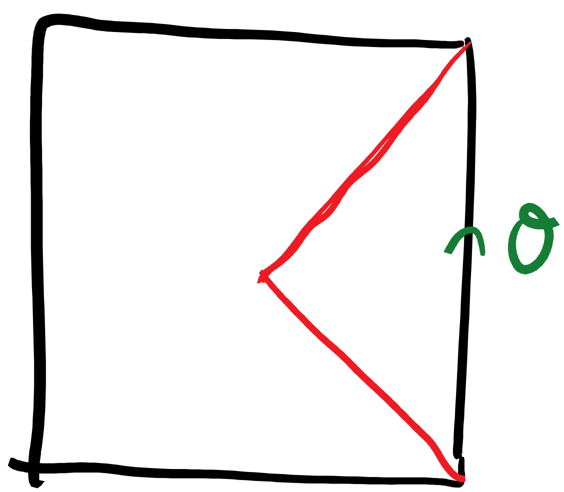


This involves finite **timelike** boundaries for *both* signs of the cosmological constant. Their consistency in EFT and in string/M theory is important to understand.

Gravitational calculations suggest an entropic interpretation of the de Sitter observer horizon area, somewhat analogous to black hole thermodynamics

Gibbons-Hawking ‘70s … Anninos Denef Law Sun ‘19 (logarithmic corrections), …

Banihashemi et al ’22 1st law via Brown-York energy



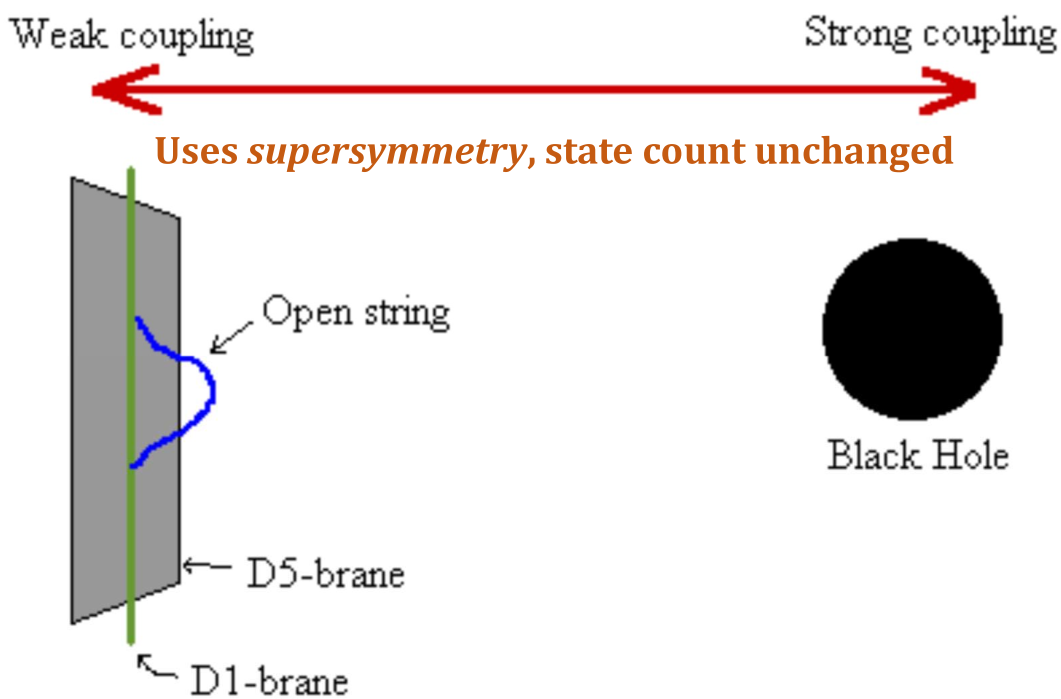
$$S = S_{GH} - 3\log(S_{GH}) + \dots$$

$S_{GH} = \frac{A}{4G_N}$

$3\log(S_{GH})$: (A) dS_3 case

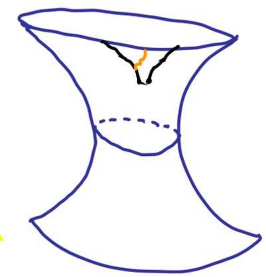
Finite number of available states, discrete quantization of energy levels, with S = log(number of available states)? Computed in some cases for black holes:

Strominger-Vafa, Callan-Maldacena,…Sen



Recent work generalizes this to *integrable deformations* to preserve state count for cosmo case.

Cosmological case:



Naively much harder because of the absence of timelike boundary for global dS => fluctuating lower-d gravity, no notion of energy

But we *can* work **with a timelike boundary** for finite (A)dS patches in GR, giving us a notion of energy (Brown-York) and non-fluctuating boundary gravity.

Moreover, $E_{\{Brown-York\}} = E_{\{dressed\ by\ T\bar{T}+\Lambda_2\}}$

Gives a holographic dual description as a deformed CFT passing nontrivial tests. -> Question: status in M theory?

$$S = S_{GH} - 3\log(S_{GH}) + \dots$$

Suggests that finite Hilbert space captures observer patch.
(cf ...Anninos/Hartnoll/Hofman ‘11...Banks et al...Susskind)

Coleman et al ’21: *Real dressed spectrum* of the universal & solvable

$T\bar{T} + \Lambda_2$ deformation

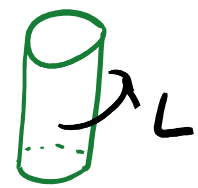
Zamalodchikov et al, Dubovsky et al, Cavaglia et al ... Gorbenko ES Torroba (GST) ’18, LLST ‘19

of a CFT on a cylinder captures the leading microstates and the radial geometry of the dS_3 observer patch.
Does *not* by itself capture all details of local bulk matter (additional specifications required for that but it’s subleading for S_{GH}=area/4G and geometry)

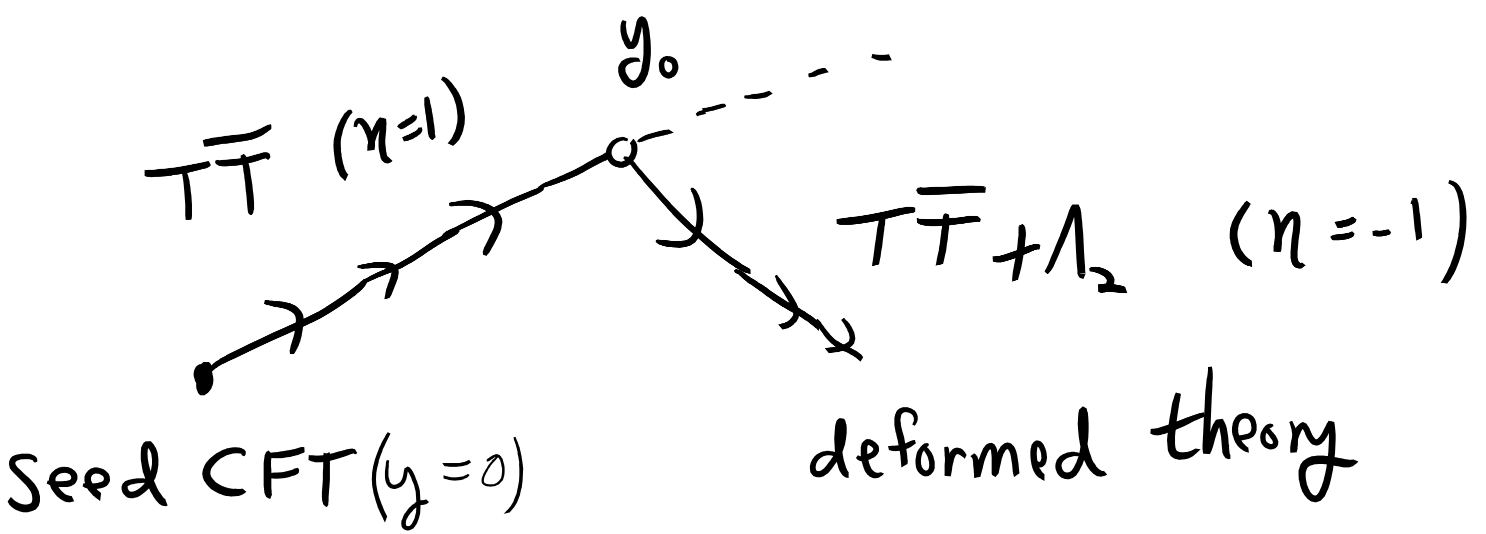
$$T\bar{T} \equiv \frac{1}{8}(T_{ab}T^{ab} - (T^a_a)^2)$$

$$\frac{\partial}{\partial \lambda} \log Z = -2\pi \int d^2x \sqrt{g}\ T\bar{T} + \underbrace{\frac{1-\eta}{2\pi\lambda^2}}_{\Lambda_2} \int d^2x \sqrt{g}$$

e.g. cylinder



$$y \equiv \frac{\lambda}{L^2}$$



Deformed energy spectrum computed precisely:

Smirnov-Zamolodchikov, Cavaglia et al, Dubovsky et al…Gorbenko et al…

$$\frac{\partial}{\partial\lambda}\log Z = -2\pi\int d^2x\sqrt{g}\;T\bar{T} + \frac{1-\eta}{2\pi\lambda^2}\int d^2x\sqrt{g}$$

$\rightarrow \partial_\lambda\langle H\rangle \sim \langle T\bar{T}\rangle,\quad T_x^x \sim \frac{\partial E}{\partial L}$ (pressure), … $\rightarrow$ $\mathcal{E} = EL$

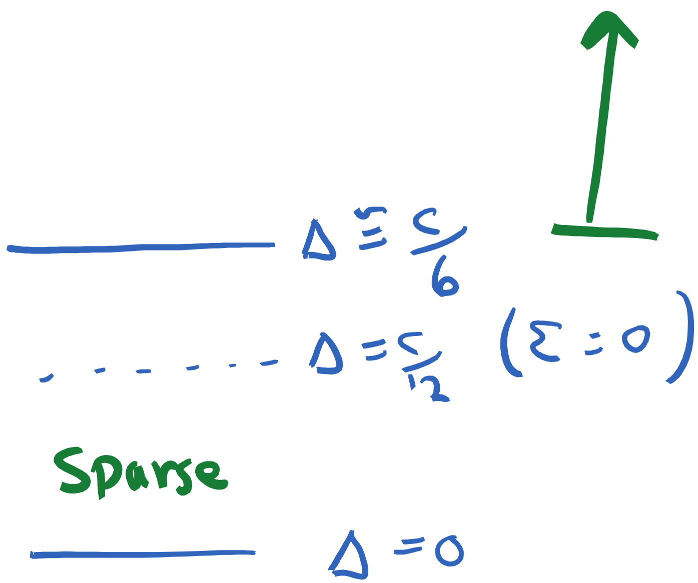
$$\pi y\mathcal{E}(y)\mathcal{E}'(y) - \mathcal{E}'(y) + \frac{\pi}{2}\mathcal{E}(y)^2 = \frac{1-\eta}{2\pi y^2} + 2\pi^3 J^2$$

$$\mathcal{E}\Big|_{y=0,\eta=1} = \mathcal{E}_{CFT} = 2\pi\left(\Delta - \frac{c}{12}\right)$$

$$\mathcal{E}\Big|_{y=0,\eta=1} = \mathcal{E}_{CFT} = 2\pi\left(\Delta - \frac{c}{12}\right)$$

We will be interested in a seed CFT with a *sparse light spectrum* (in particular a holographic CFT)

Hartman Keller Stoica et al



$$S = S_{Cardy} = 2\pi\sqrt{\frac{c}{3}\left(\Delta - \frac{c}{12}\right)}$$

$\simeq \frac{A}{4G_N}$ for holographic CFTs, with BTZ Black Holes for $\Delta > \frac{c}{12}$

General solution:

$$\mathcal{E}(y) = \frac{1}{\pi y}\left(1 \pm \sqrt{\eta - 4C_1 y + 4\pi^4 J^2 y^2}\right)$$

Fix integration constant and branch via appropriate boundary conditions for a given trajectory in theory space.

We will do two key examples, where the deformed energy matches the *Brown-York energy* for a given patch of dS, via a trajectory which is continuous for the corresponding band of energies.

Brown-York (quasilocal) stress-energy

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta I_{\text{on-shell}}}{\delta g^{\mu\nu}} = \frac{1}{8\pi G_N}\left(K_{\mu\nu} - g_{\mu\nu}K + \frac{1}{\ell}g_{\mu\nu}\right)$$

$\sqrt{\ldots}$ Part of $\mathcal{E}$

With cylinder slices, a subset of the Einstein equations imply the above differential equation for $\mathcal{E} = L\int_0^L dx\, T_t^t$

with dictionary $$c = \frac{3\ell}{2G_N}, \quad \lambda = 8G_N\ell\,. \quad \Lambda_3 = -\frac{\eta}{\ell^2}$$

McGough, Mezei, Verlinde; Kraus, Liu, Marolf, Kraus, Monten, Roumpedakis, Ebert, Hijano, Caputa, Datta, Jian, Myers,… , Gorbenko ES Torroba,… Banihashemi, Jacobson, Vissar…1st law of thermodynamics wrt $E_{Brown-York}$.

*Dirichlet boundaries not fully understood:

- Certain potential instabilities Marolf/Santos et al
- Difficulties defining perturbation theory Anderson, Witten et al
  exceptions for definite-sign extrinsic curvature, also UV-sensitive
- Generalization to UV complete theory? In progress (below)

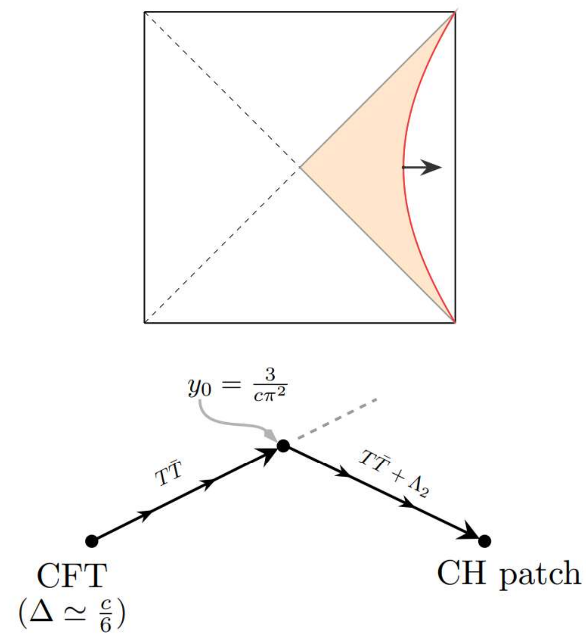
*$T\bar{T}$ theories not fully understood:

- Full set of observables?

Other boundary conditions also possible in this framework Coleman-Shyam, different ensembles. Bounded regions can be building blocks for joined system.

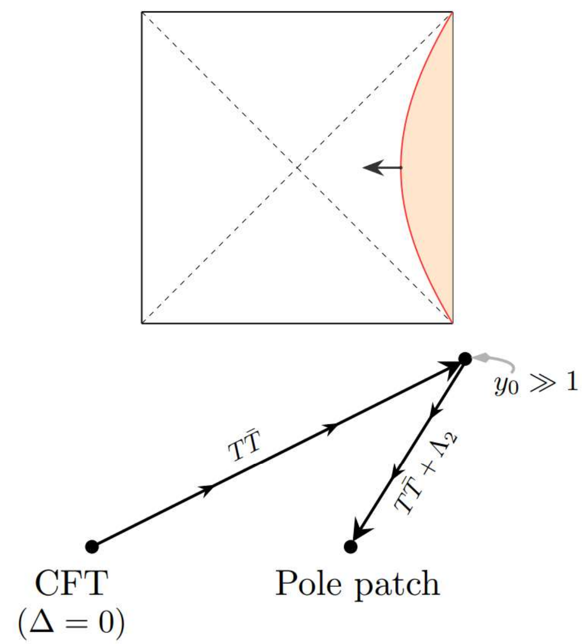
Cosmic horizon patch

(Dressed $\Delta \simeq \frac{c}{6}$ black hole microstates)



Pole patch

(Dressed $\Delta = 0$ vacuum)



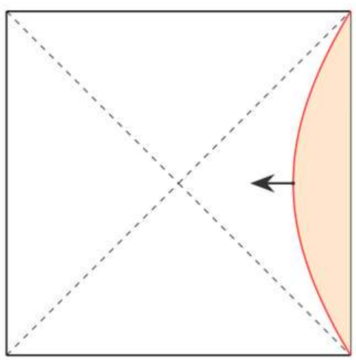
$\mathcal{E} = \frac{1}{\pi y}\left(1 + \sqrt{\eta + \ldots}\right)$ $\longleftarrow$ related by $\pm\sqrt{\ }$ $\longrightarrow$ $\mathcal{E} = \frac{1}{\pi y}\left(1 - \sqrt{\eta + \ldots}\right)$

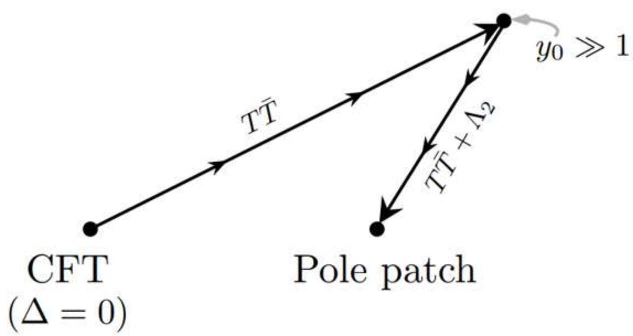
$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$

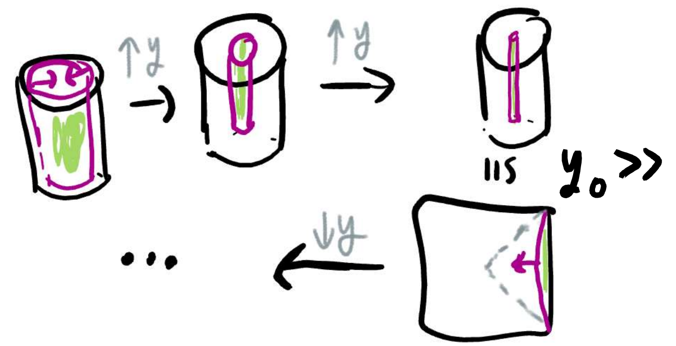
Pole patch

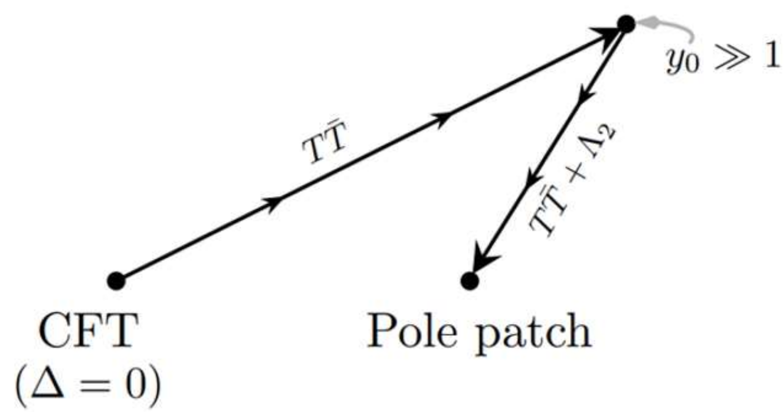
(Dressed $\Delta = 0$ vacuum)

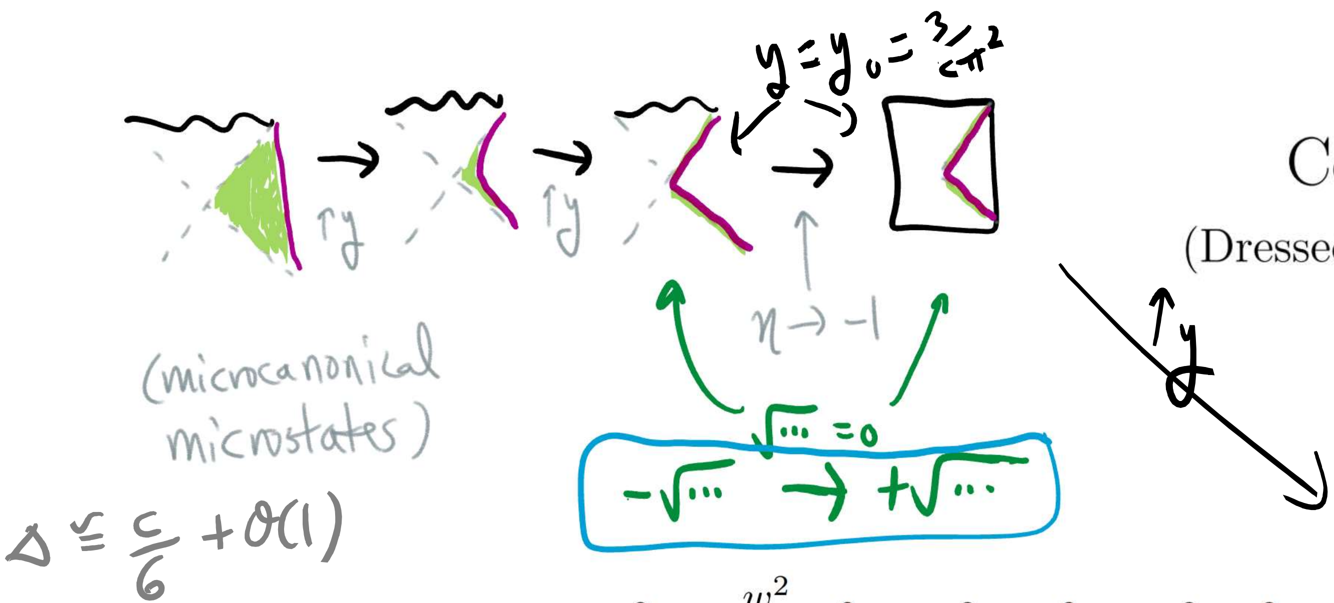
Gorbenko ES Torroba ’18
Lewkowycz Liu ES Torroba ‘19





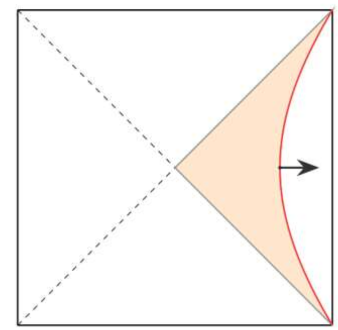






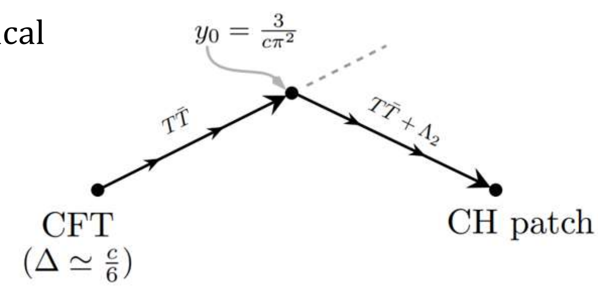
Cosmic horizon patch

(Dressed $\Delta \simeq \frac{c}{6}$ black hole microstates)



$$ds_3^2 = -\frac{w^2}{\ell^2}d\tau^2 + dw^2 + (\ell^2 + \eta w^2)d\phi^2$$

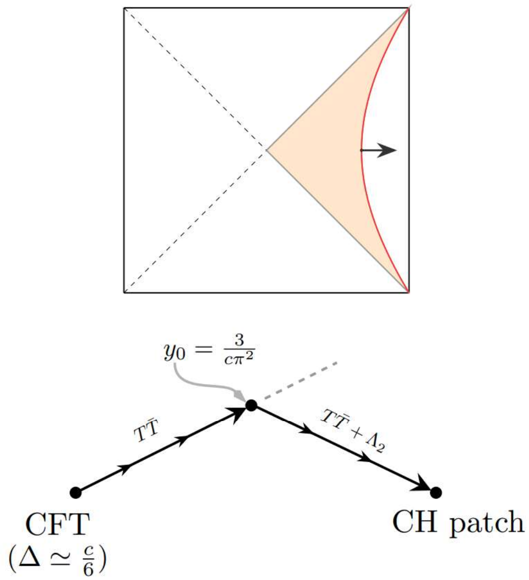
At $y = y_0$, the near horizon patches are identical



$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$

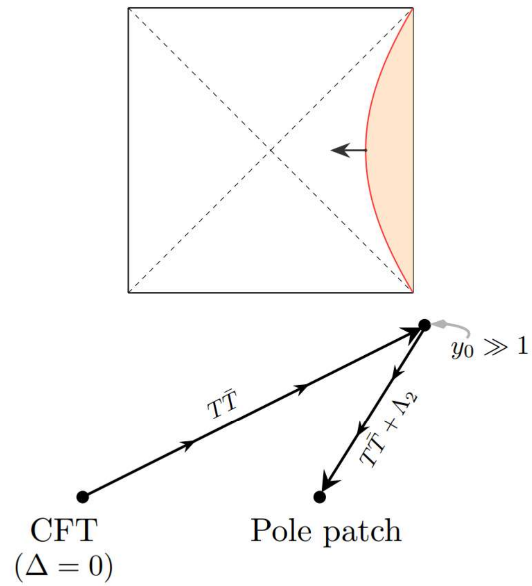
Cosmic horizon patch

(Dressed $\Delta \simeq \frac{c}{6}$ black hole microstates)



Pole patch

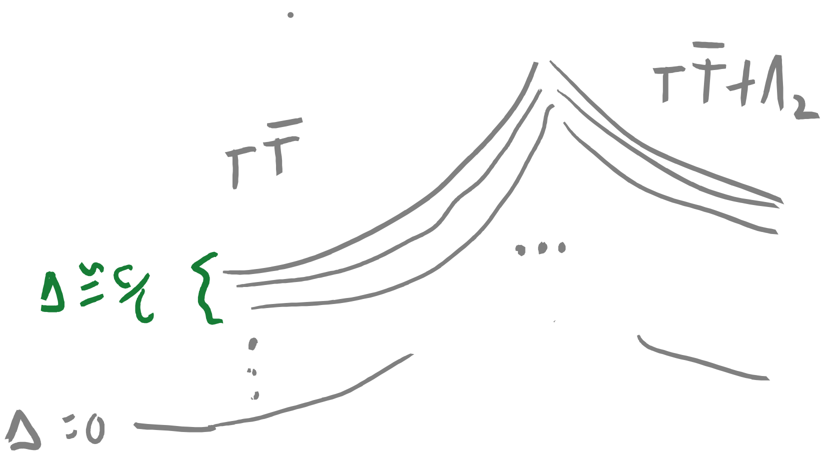
(Dressed $\Delta = 0$ vacuum)



$\mathcal{E} = \frac{1}{\pi y}\left(1 + \sqrt{\eta + \ldots}\right)$ $\longleftarrow$ related by $\pm\sqrt{}$ $\longrightarrow$ $\mathcal{E} = \frac{1}{\pi y}\left(1 - \sqrt{\eta + \ldots}\right)$

As we vary y, we capture precisely the geometry of the dS patch

$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$



Count of states goes along for the ride (`integrable deformation').
Note: follow states w/o using SUSY BPS arguments

$$\Delta < \frac{c}{12} : \text{ sparse spectrum } \quad \text{(particle states)}$$

$$\Delta \geq \frac{c}{6} : S \simeq S_{Cardy} = 2\pi\sqrt{\frac{c}{3}\left(\Delta - \frac{c}{12}\right)}$$

Other states:

$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$

$\Delta > \frac{c}{6}$ states: dressed energies formally become complex, discarded in a unitary version of the theory => Finite dimensional Hilbert space with count of states agreeing with Gibbons-Hawking

$\Delta < \frac{c}{6}$ states: subdominant at large c, model-dependent (details require additions to the deformation). Includes interesting landscape states.

=> Real spectrum of the $T\bar{T} + \Lambda_2$ deformation captures the finite dimensional Hilbert space (i) agreeing with Gibbons-Hawking and (ii) building up the geometry of the dS observer patch

Count of states goes along for the ride (`integrable deformation'), subleading check agrees at `pure gravity' level:

$$S = A/4G_N - 3\log(A/4G_N)$$

Sen (AdS BTZ case) … Anninos Denef Law Sun (dS)

Cf Bousso-Maloney-Strominger '01 Kerr-dS entropy from Cardy formula;
In some sense this explains that (self-described) `numerology'.
Cf van Leuven, E. Verlinde, Visser '18, DST '18

Summary: At the `pure gravity' level, the *real dressed spectrum* of the universal and solvable

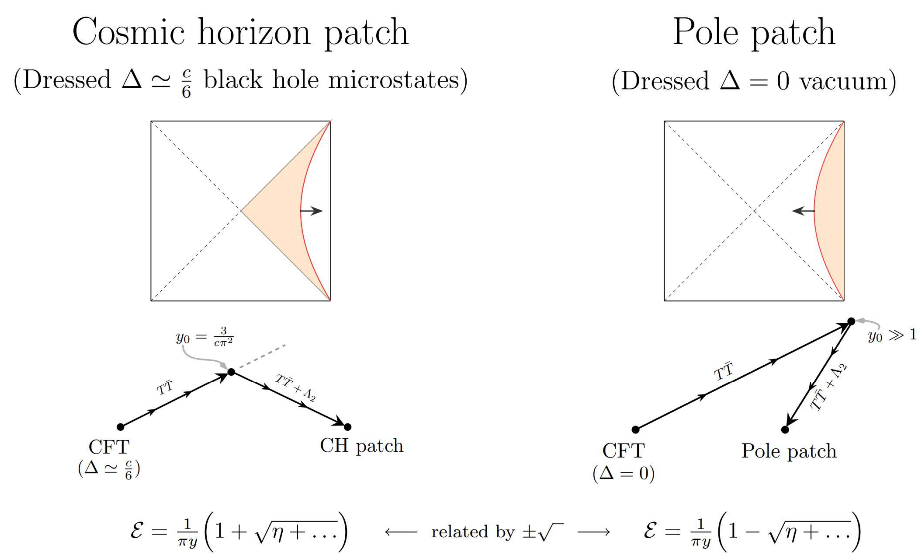
$$\frac{\partial}{\partial\lambda}\log Z = -2\pi\int d^2x\,\sqrt{g}\,\langle T\bar{T}\rangle + \frac{1-\eta}{2\pi\lambda^2}\int d^2x\,\sqrt{g}$$

$T\bar{T} + \Lambda_2$ deformation

Zamalodchikov et al, Dubovsky et al, Cavaglia et al … Gorbenko ES Torroba '18

of a CFT on a cylinder captures (only) the microstates and the geometry of the dS_3 observer patch Shyam, Coleman et al '21

$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$



BPS black hole state counting (Strominger/Vafa), used extended SUSY to control weak → strong coupling deformations preserving state count. Here we have a **new type of controlled deformation** applicable to dS, again preserving state count: 'integrable deformation' of non-integrable seed theory.

Further generalizations of $T\bar{T} + \Lambda_d$:

M. Taylor; Guica e tal, Hartman Kruthoff Shaghoulian Tajdini '18, Shyam et al …

(1) Local bulk matter (model-dependent, subleading in entropy) requires similar term for each low-energy field: (large c regime). Use for $\Delta < c/6$ states. De Luca et al in progress

$\Pi_{\phi,\text{radial}} \sim \mathcal{O} \rightarrow$

$$T^i_i = -4\pi\lambda T\bar{T} - \frac{1}{\pi\lambda}\left(\frac{c\lambda}{L^2}\right)^{\Delta}\tilde{\mathcal{O}}^2 - \frac{cR^{(2)}}{24\pi} + \Lambda_2$$

(3) More general finite-c solvable deformation, with bulk gauge field matter:

$$T\bar{T} + \frac{J\bar{J}}{\lambda} + \frac{1-\eta}{\lambda^2}$$

Batra et al in progress

(2) Higher dimensions & curvature (large c regime):

$$\tilde{T}^\mu_\mu = -4\pi G\ell\left(\tilde{T}_{\mu\nu}\tilde{T}^{\mu\nu} - \frac{1}{d-1}(\tilde{T}^\mu_\mu)^2\right) - \frac{\ell}{16\pi G}R^{(d)} - \frac{d(d-1)}{16\pi G\ell}(\eta - 1)$$

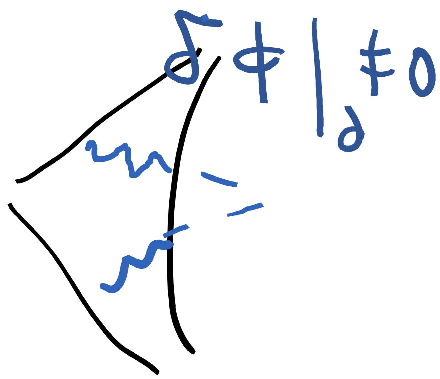
$$\tilde{T}_{\mu\nu} \equiv T_{\mu\nu} + a_d C_{\mu\nu}\,. \qquad C_{\mu\nu} = G_{\mu\nu} \text{ for } d \leq 4$$

So far, at least without "OO" deformation, we have a discrete, finite quantum mechanics system. Type I algebra Cf type II in Chandrashekharan, Penington, Witten.

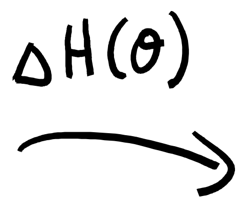
With the extra deformation needed for local bulk matter, say in the Q.M. version where the composite ops are easy to define, do these properties continue? Not known for sure, we might expect so (?): Work at 1 << c < ∞.

Without OO's (pure $T\bar{T}+\Lambda$)

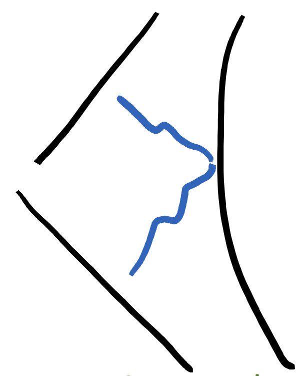
Finite and discrete spectrum, spacings ~ exp(-S)

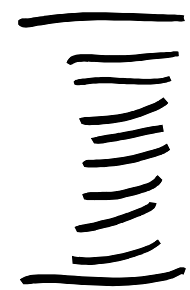


Gravity side picture: ΔH effects a change of ϕ boundary conditions to achieve ~bulk locality.



$T\bar{T}+\Lambda+OO$





Some other finite and discrete spectrum (?) Otherwise, ΔH would need to bring in an infinite number of states from somewhere to fill in a continuum.

- tOmperature $\sim \frac{1}{\beta}$ from $\partial_c E_{dressed}$ Lin/Susskind: from above dictionary and

$$ds_3^2 = -\left(\frac{\ell^2 - r^2}{\ell^2}\right) d\tau^2 + \left(\frac{\ell^2}{\ell^2 - r^2}\right) dr^2 + r^2 d\phi^2$$ we get $$(\beta/L)^2 = \frac{y}{y_0} - 1$$

$$\mathcal{E} = \frac{1}{\pi y}\left(1 \mp \sqrt{\eta + \frac{y}{y_0}(1-\eta) - 4\pi^2 y\left(\Delta - \frac{c}{12}\right) + 4\pi^4 y^2 J^2}\right)$$

-1

$\partial_c E_{dressed} \propto 1/\beta$. Change in energy upon changing # d.o.f.

- Complexity: note that we cannot expand the square root once $\eta = -1$. If we take the seed CFT as proxy for the `fundamental' degrees of freedom and $E_{dressed}$ as proxy for the Hamiltonian, it means we have fully nonlinear all to all interactions in the case (dS) where complexity blows up.

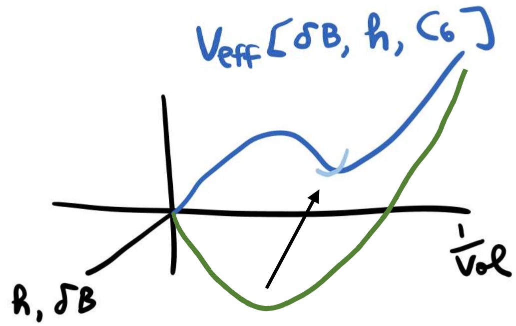
This is encouraging, but raises many questions

- Generalization to 4-dimensional dS? cf Hartman et al, Shyam,…

- Relation to string theoretic de Sitter (=dS quantum gravity)?

Late time physics (metastable decay)?

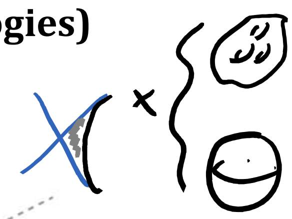
M/String theory includes direct uplifts from AdS/CFT Dong et al '10, De Luca et al '21, again connecting the dS case to a CFT. e.g. recent example:

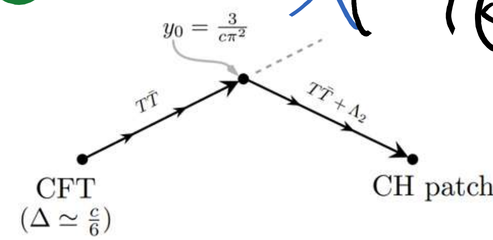


dS x hyperbolic space 

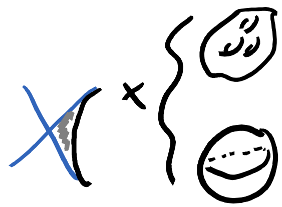
(Very different internal topologies)

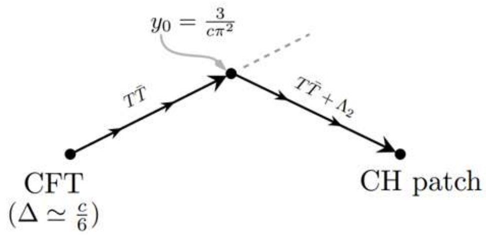
AdS x sphere 





Nonetheless, the matching between BH and dS horizons may extend to the full UV theory





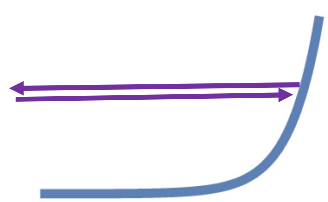
The matching happens at large \`tOmperature' of the boundary theory

=> Mixing among all internal configurations consistent with the horizon. Even in the full string/M theory, can't distinguish the AdS/BH and dS horizons.

This relies on the existence of the bounding walls => question:
do they exist in full quantum gravity (string/M theory)? In terms of embedding of the fundamental degrees of freedom into the target spacetime:

$$Matrix\ Theory\ Hamiltonian = \sum Tr(\dot{X}^2) + Tr[X^M, X^N]^2 + \mathrm{Tr}\ O_\kappa \exp(\kappa X^{(10)})$$

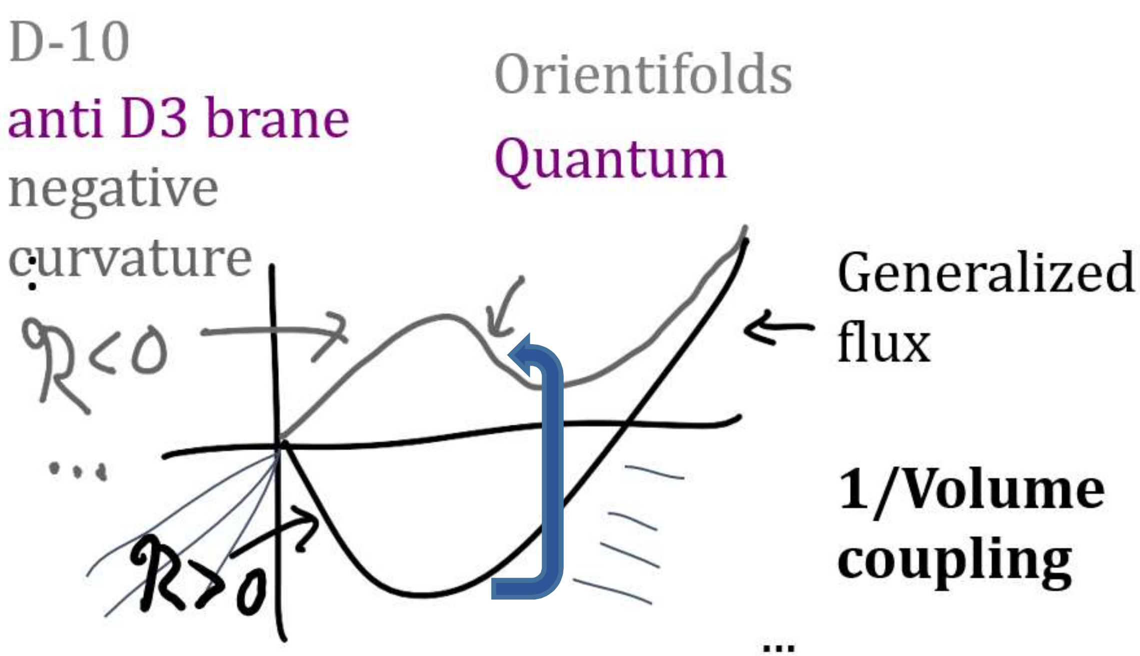
$$\text{String theory worldsheet action} = tension * \int (G_{MN}\partial X^M \partial X^N + O_\kappa \exp(\kappa X^{(9)}))$$

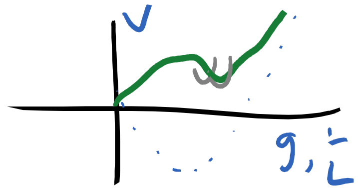


The problem in string/M theory:

Again, the matching occurs at the horizon in the **external** dimensions
(AdS BH horizon ≃dS cosmic horizon)

Uplift from AdS/CFT to dS: nontrivial **internal topology change





4d effective potential

Douglas '09

Mostly positive:
$D - D_c, -R^{D-4}, (Q_1 + a\, Q_2)^2, \ldots$
Intermediate negative:
O-planes, quantum

$$V_{eff}[g^{(D-4)}, \ldots] = \frac{\ell_D^{D-2}}{2G_N^2} \frac{\int d^{D-4}y\, \sqrt{g^{(D-4)}} e^{-2\Phi} u^2|_c \left(-R^{(D-4)} - \frac{1}{4}\ell_D^{D-2} T^\mu_\mu - 3\left(\frac{\nabla u}{u}\right)^2 \Big|_c \right)}{\left(\int d^{D-4}y \sqrt{g^{(D-4)}} e^{-2\Phi} u|_c\right)^2}$$

Net curvature

$u(y) = e^{2A(y)}$

$$ds^2 = e^{2A(y)} ds^2_{dS_4} + e^{2B(y)} (g_{\mathbb{H}ij} + h_{ij}) dy^i dy^j$$

$R_{sec} < 0$ rigid
(cf Trodden et al, Saltman-ES, DLST)

$R_{ij} = 0$ CY
(cf KKLT, LVS...)

u(y) satisfies GR constraint (its eq. of motion):

$$\left(-\nabla^2 - \frac{1}{3}\left(-R^{(D-4)} - \frac{1}{4}\ell_D^{D-2} T^\mu_\mu \right) \right) u = -\frac{C}{6}$$

Like a Schrodinger problem for
$C\ell^2 \sim H^2\ell^2 \ll 1$

$$\Rightarrow \quad V_{eff} = \frac{C}{4G_N} = \frac{R^{(4)}_{\rm symm}}{4G_N}.$$

Warp factor stabilizes runaway negativity (e.g. $-B'^2$)

dS examples stabilizing extra dimensions:

Reviews of various aspects: Polchinski, Baumann/McAllister, Douglas/Kachru, Denef, Frey, Hebecker; ES TASI '16, ...

- Non-perturbative stabilization

--GKP '01/KKLT '03 and many followups, e.g.
--large volume scenario

Sub-KK scale SUSY breaking

- Power-law stabilization

--(D-Dc), O-planes, flux, asymmetric orbifold (large-D expansion) '01-'02
(...other examples...)
--hyperbolic space, Casimir, flux '21

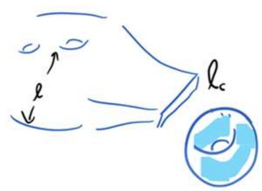
-- RG logs & powers Burgess/Quevedo '22

--including explicit uplifts of AdS/CFT
[D1-D5 theory -> dS3 '10,
M2 brane theory -> dS4 '21]

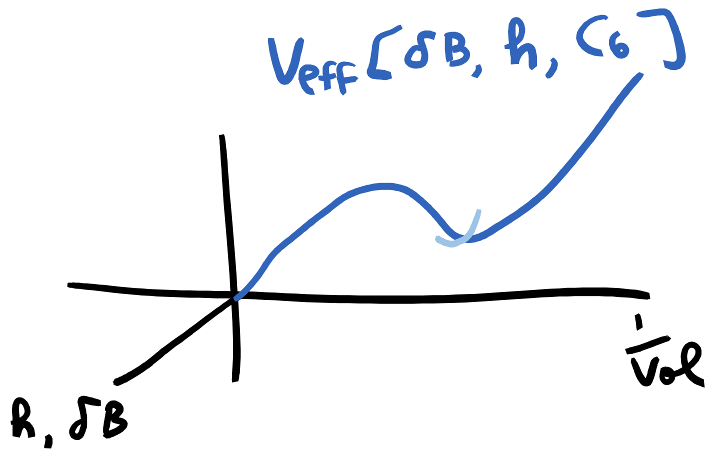
≥KK scale SUSY breaking

Weak-coupling EFT/large-N/Large-D/small W_0 control.
Ongoing studies of internal equations of motion in various cases & models, including ones with significant gradients e.g. Cordova et al, ...

Curved internal dim's: recent mechanism for Λ from string/M theory



M theory (EFT: 11d SUGRA) on explicit infinite discrete family of finite-volume hyperbolic spaces with $\int -R - 3u'^2 \ll -\int R$ **parametrically**, automatically-generated Casimir energy, 7-form flux yields immediate volume stabilization and approximate piecewise solution dressed with warp & conformal variations, small residual tadpoles.



Strong positive Hessian contributions from **hyperbolic rigidity** and from **warping** (redshifting) effects on conformal factor and on Casimir energy.

Douglas '09

4d effective potential

net curvature term

$\ell_{11}^9 \rho_c(R_c) \sim -\frac{\ell_{11}^9}{R_c^{11}}$

$$V_{eff}[g^{(7)}, C_6] = \frac{\ell_{11}^9}{2G_N^2} \frac{\int d^7y \sqrt{g^{(7)}} u^2|_c \left([-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 \Big|_c] - \frac{1}{4}\ell_{11}^9 T^{(Cas)\mu}_{\ \ \mu} + \frac{1}{2}|F_7|^2 \right)}{(\int d^7y \sqrt{g^{(7)}} u|_c)^2}$$

$$ds^2 = e^{2A(y)} ds^2_{dS_4} + e^{2B(y)}(g_{\mathbb{H}ij} + h_{ij})dy^i dy^j \qquad u(y) = e^{2A(y)}$$

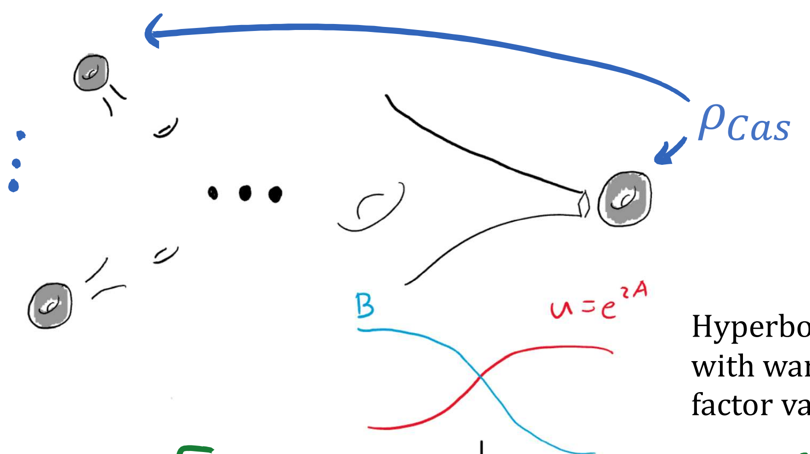
u(y) satisfies GR constraint (its equation of motion):

$$\left(-\nabla^2 - \frac{1}{3}\left(-R^{(7)} - \frac{1}{4}\ell_{11}^9 T^{(\mathrm{Cas})\mu}_{\ \ \mu} + \frac{1}{2}|F_7|^2\right)\right) u = -\frac{C}{6}$$

Like a Schrodinger problem for

$C\ell^2 \sim H^2\ell^2 \ll 1$

$$\Rightarrow \quad V_{eff} = \frac{C}{4G_N} = \frac{R^{(4)}_{\text{symm}}}{4G_N}.$$



Hyperbolic manifold dressed with warp and conformal factor variations

Tune small to compete with Casimir with $\ell_{11} \ll R_c \ll \ell$ →

$$\left[-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 < 0 \;\middle|\; -R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 > 0 \right]$$

warp & conformal factor eoms ⇒

$$-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 = 4\ell_{11}^9 |\rho_C| - \frac{C}{u} - \frac{5}{2}F_7^2$$

Douglas Kallosh '10

$$a = \frac{\int \sqrt{g^{(7)}} u^2|_c [-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 \Big|_c]}{\int \sqrt{g^{(7)}} u^2|_c \, 42/\ell^2} \ll 1 :$$

$$-R^{(7)} - 3\left(\frac{\nabla u}{u}\right)^2 = 4\ell_{11}^9 |\rho_C| - \frac{C}{u} - \frac{5}{2} F_7^2$$

Balance Terms in U => $\hat{\ell}^4 \sim \frac{1}{\hat{\ell}_c^5} \frac{n_c}{v_7} \frac{Vol(T^6)}{\lambda_c^6} \cdot \frac{1}{a}$

If a sufficiently small, then all length scales large:

$$\ell \gg \ell_c \gg \ell_{11}$$

- If *a* is too large, increase volume of non-Casimir regions (e.g. via short filled cusps or covers k-fold -> (k+1)-fold)

- If *a* is too small, reduce flux quantum number

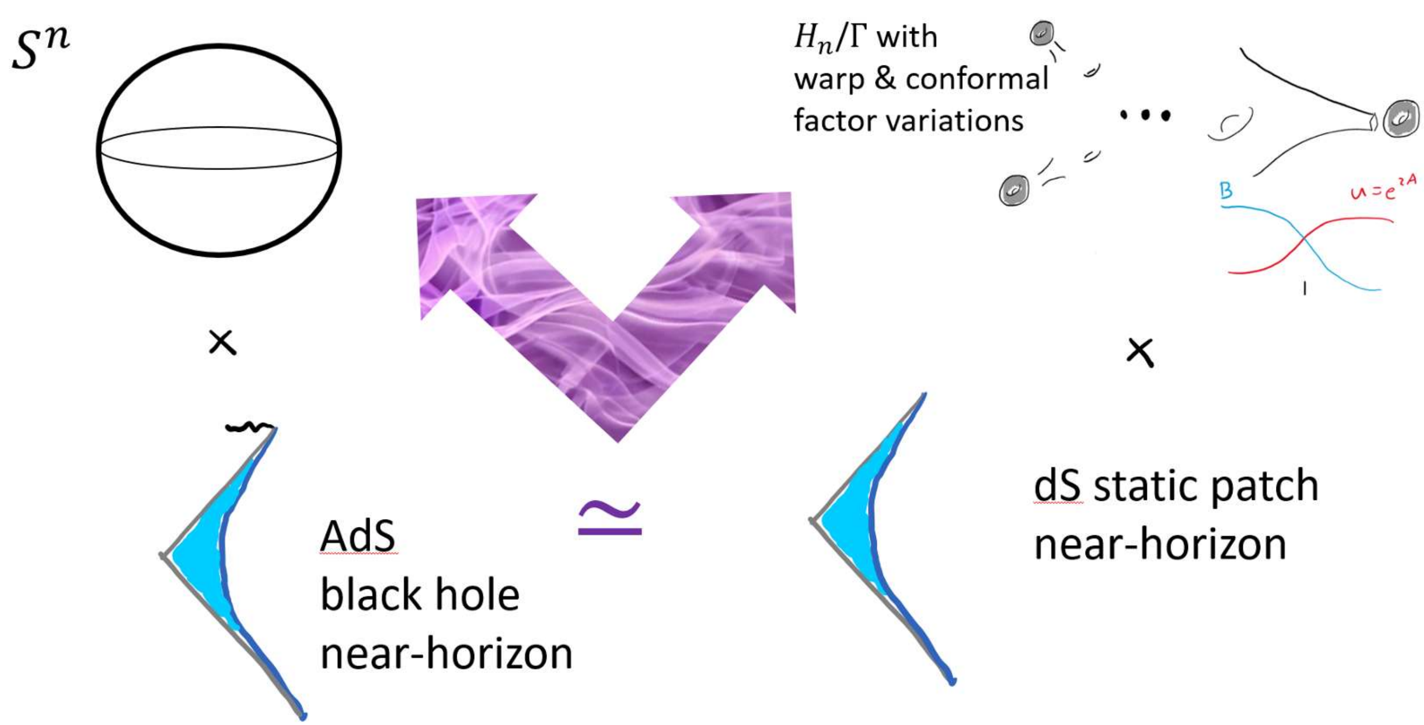
Work with simple concrete hyperbolic manifolds with comparable cusp and bulk volumes Italiano et al '20. Explicit radial solution illustrates $a \ll 1$.

Parametric suppression of residual tadpoles.

...Coming back to our problem:

The matching point corresponds to a high-temperature boundary (canonically)
=> fluctuates among all internal configurations consistent with the horizon,

So also can't tell the difference between dS and AdS/BH **internally**!

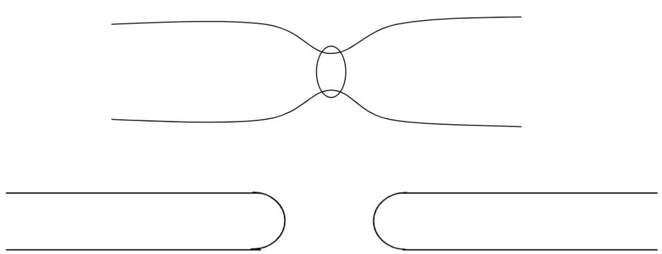


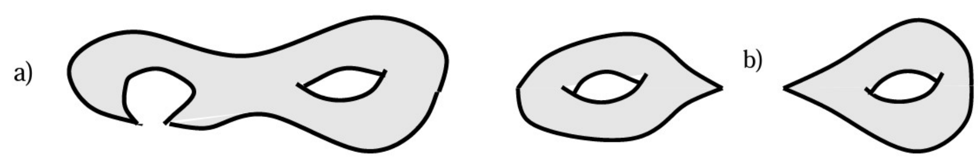
This assumes:

(1) The requisite (topology, D, …)-changing processes are possible

Many precedents:

Conifold transitions, change of Riemann surface genus, chirality-changes, dimension-changes via condensation of wrapped branes/strings:





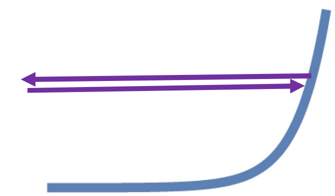
(2) The timelike boundaries exist in M/string theory

(2) Do the timelike boundaries exist in M/string theory?

Let's take the approach of generalizing Liouville walls:

$$ds^2 = e^{2A(y)} \left\{ d\chi^2 + f_1(\chi)(-dt_{||}^2 + f_2(\chi)(d\vec{x}_{||})^2) \right\} + g_{mn}(y) dy^m dy^n$$

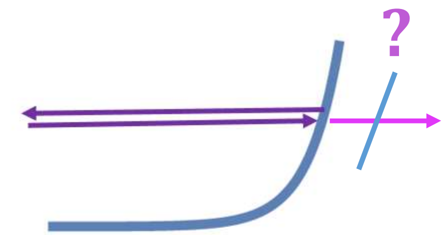
$$S_{ws} = S_{ws}^{(0)} + \int_\Sigma [\hat{O}_\Delta \Phi]_r \sim \int d^2\sigma \hat{O} e^{\kappa\chi} \qquad \chi \gg 1/\kappa$$



$\Delta > 2$

Deform the semiclassical worldsheet action by marginal, i.e. dimension (1,1), massive vertex operator. Then check that the worldsheet path integral has no support at $\chi \to \infty$. Here this is nontrivial, depending on $\hat{O}$.

Want to check that worldsheet cannot ooze out to ∞



Flat target: $$ds^2 = -dX^{0\,2} + \sum_{i=1}^{D-1} dX^{i\,2}$$

$$S_{ws} = S_{ws}^{(0)} - \lambda_W \int_\Sigma [\{(\partial_- X_\parallel)^2(\partial_+ X_\parallel)^2\}^2 e^{\kappa X_{D-1}}]_r$$

Marginal => $\kappa = \sqrt{\frac{2\Delta - 4}{\alpha'}} = \frac{2\sqrt{3}}{\sqrt{\alpha'}}$

from $$S_{ws}^0 - \frac{1}{4}\lambda_W \int d^2\sigma\sqrt{-h}\{(J_\parallel^2)_{\alpha\beta}(J_\parallel^2)_{\gamma\delta}(h^{\alpha\gamma}h^{\beta\delta} - \frac{1}{2}h^{\alpha\beta}h^{\gamma\delta})\}^2 e^{\kappa X^{D-1}}$$

We show: $$\hat{O}_8 = [(\partial_- X_\parallel)^2(\partial_+ X_\parallel)^2]^2 \not\to 0 \quad as \quad X^{D-1} \to \infty$$

We see that there is indeed no transmission, as follows: $(\partial_+ X)_\parallel^2 \neq 0$, it can't vanish: it's related by the worldsheet constraints $\delta_h S_{ws} = 0$ to $(\partial_+ X)_{\{D-1\}}^2$. The latter can't vanish for a string propagating to $X_{\{D-1\}} \to \infty$.

Similarly, vacuum
NS-NS $AdS_3 \times S^3 \times X_4$

Maldacena/Ooguri, Kutasov Seiberg et al, …

$$ds^2 = d\rho^2 - \cosh^2\frac{\rho}{\ell}dt^2 + \sinh^2\frac{\rho}{\ell}d\phi^2 + internal$$

$$\Delta S_{ws} = -\lambda_W \int d^2\sigma(\hat{O}_R + T_{++,internal})^2(\hat{O}_L + T_{--,internal})^2\Phi_{m^2}(\rho, t, \phi)$$

$u, v = \frac{1}{2}(t \pm \phi)$ parallel to boundary, Φ_{m^2} solution to massive wave equation

$$\hat{O}_R = -(J_R^3)^2 = -k^2(\partial_+ u + \cosh(2\rho)\partial_+ v)^2$$

Again here the string cannot ooze out to infinity for any path integral configuration, implying a wall.

Moreover, there is a net Brown-York energy and string charge (NS-NS flux), suggesting consistent with an effective Dirichlet condition at the wall.

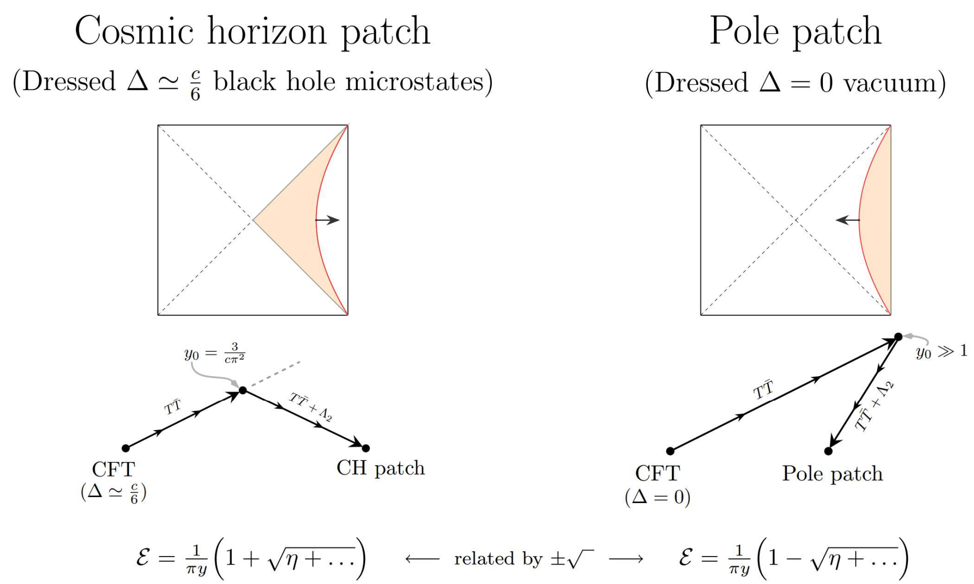
The internal $S^3 \times X_4$ is constant radially, consistent with the possibility of a fluctuating boundary condition for them (required for the melting at the matching point).

Our problem of interest requires generalization to:

- AdS black holes: fewer symmetry currents J
- dS: Fischler-Susskind worldsheet
- M theory (11d) case: M(atrix) theory interpolation between 11d and 10d M(atrix) string theory, e.g. in flat space

$$H = H_0 + Tr[\{(D_- X_\parallel - \frac{1}{4}\psi_\parallel \kappa \psi^D - 1)^2 (D_- X_\parallel - \frac{1}{4}\psi_\parallel \kappa \psi^D - 1)^2\}^2 e^{\kappa X^{10}}]$$

In general, the open question of finite (non-asymptotically AdS) timelike boundaries in general spacetimes is key for holography, e.g. dS

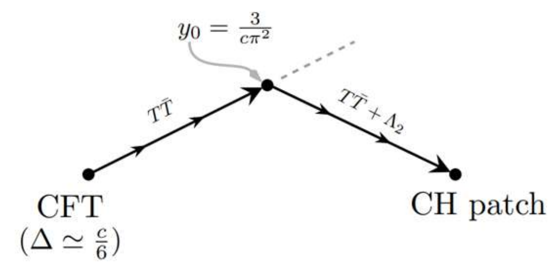


Summary:

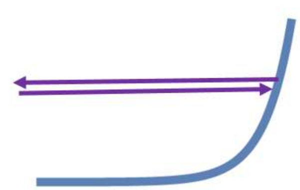
- Solvable deformations capture the geometry and microstate count of the dS static patch, via integrability of the deformation. Extends to

$$T\bar{T} + \frac{J\bar{J}}{\lambda} + \frac{1-\eta}{\lambda^2}$$

- Raised the question of how this could embed in string/M theory, given the enormous difference between the internal spaces for AdS and dS. This is answered automatically by the fact that the matching between the $\eta = \pm 1$ trajectories ($y = y_0$) occurs at the horizon, where internal thermal averaging is compulsory.

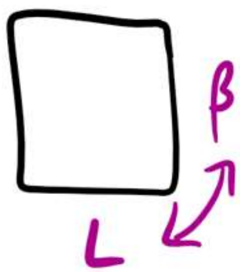


- Timelike boundaries in string/M theory may arise from generalizing Liouville walls to (super-)critical D with $\Delta S_{ws} \sim \int \hat{O} \exp(\kappa X_{radial})$



Extra Slides

In the canonical ensemble (fixed `tOmperature' $\sim 1/\beta$ and L: Euclidean torus boundary), our system exhibits an intriguing remnant of modular invariance



A 2d theory on a torus is invariant under $\beta \leftrightarrow L$ $\quad Z_\lambda(L,\beta) = Z_\lambda(\beta, L)$

But our theory, without the complex levels, is a 1d (quantum mechanics) theory, unitary but not fully local. Nonetheless, we find a remnant of modular invariance:

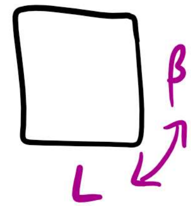
Seed CFT for c >> 1: $\log Z \simeq \max\{\underbrace{-\beta E_{vac}(L)}_{\beta > L}\ ,\ \underbrace{-L E_{vac}(\beta)}_{\beta < L}\}$ Hartman Keller Stoica et al

Deformation (β<L) : $\log Z|_{\beta<L} \simeq -LE_{vac}(\beta) = S_{Cardy}(\Delta = c/6) - \beta E_{\Delta = c/6}(L)$

The deformed $\Delta \simeq c/6$ levels propagate in the direct channel.

Shyam '21: this modular transformation starting from the pole patch spectrum yields S_{GH}

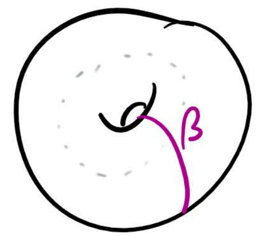
Relation/analogy to Hawking - Page :



AdS : $T < T_{HP}$

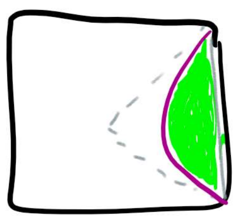
$\tilde{L} \to 0$ in interior

$T > T_{HP}$

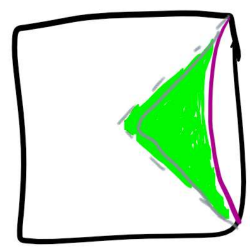


$\tilde{\beta} \to 0$ ($\tilde{T} \to \infty$) in interior

dS :

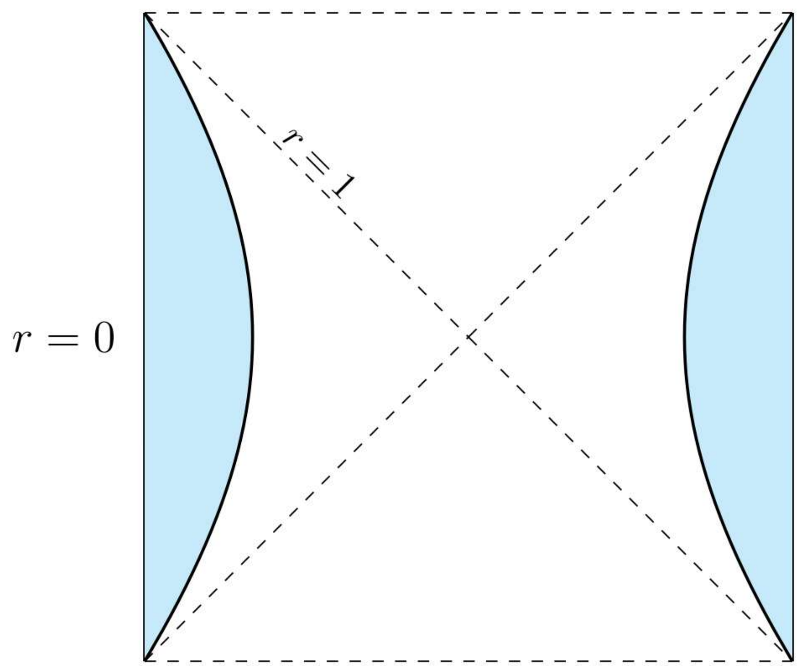


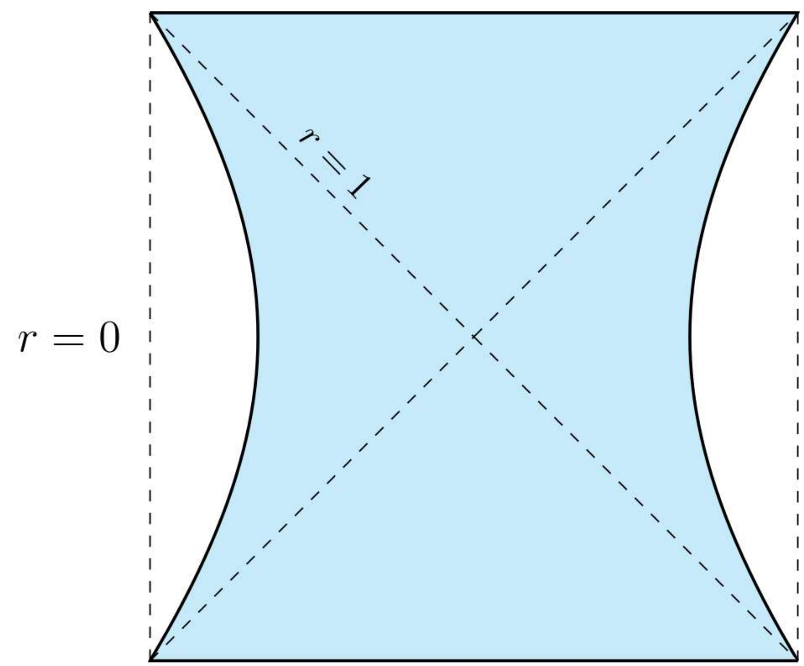
$\tilde{L} \to 0$ in interior



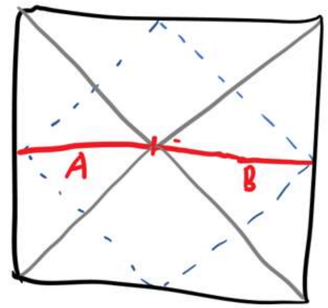
$\tilde{\beta} \to 0$ ($\tilde{T} \to \infty$) in interior

Can in principle double and glue the two patches together to get global dS:





Next: Maximal Mixing review



Black: global dS
Blue: dS/dS patch
Grey: static patches

It is interesting to consider global dS as a purification of the static patch. There is a path integral saddle corresponding to the VN and Renyi entropies for a division into two halves A and B (sensible at least near large c, semiclassically on the gravity side). This results in maximal mixing, confirmed by several independent calculations. DST, LLST. dS/dS gives a natural physical interpretation of this division so we'll also review it.

*General theory (Lewkowyzc/Maldacena '13, Dong '16): $Tr\ \rho_A^n$: replicate the spacetime according to the desired division. With gravity, smooth out the surfeit angles. Orbifold by Z_n, introducing a cosmic brane C_n which back reacts (including gravity where dynamical). The area of the cosmic brane in the saddle gives the Renyi entropy:

$$-n^2\partial_n\left(\frac{1}{n}\log\operatorname{Tr}\rho_1^n\right) = \tilde{S}_n = \frac{A(C_n)}{4G_{d+1}}$$

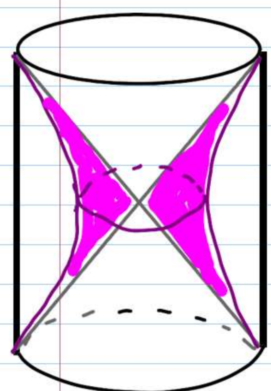
*Donnelly/Shyam '18, LLST: Entropies (up to shift) from dressed stress-energy.

$$L\frac{d}{dL}\log Z_n = -\int d^2x\sqrt{g}\,\langle\operatorname{tr}T\rangle\ ,\ L\frac{d}{dL}\tilde{S}_n = -(1-n\partial_n)\int d^2x\sqrt{g}\,\langle\operatorname{tr}T\rangle\ .$$

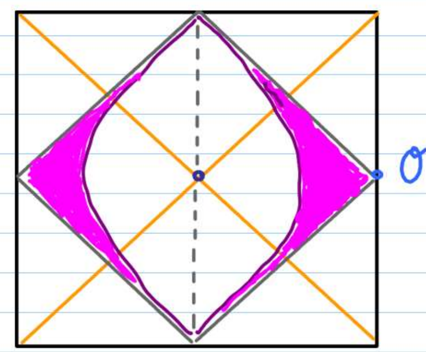
dS/dS:

Alishahiha et al ‘04, …, Dong ES Torroba ‘18, …
Gorbenko ES Torroba ’18, Shyam ‘21

$$
\begin{aligned}
ds^2_{(A)dS_{d+1}} &= dw^2 + \sin(h)^2(\frac{w}{\ell_{dS}})ds^2_{dS_d} \\
&= dw^2 + \sin(h)^2(\frac{w}{\ell_{dS}})\left[-d\tau^2 + \ell^2_{dS}\cosh^2\frac{\tau}{\ell_{dS}}d\Omega^2_{d-1}\right].
\end{aligned}
$$



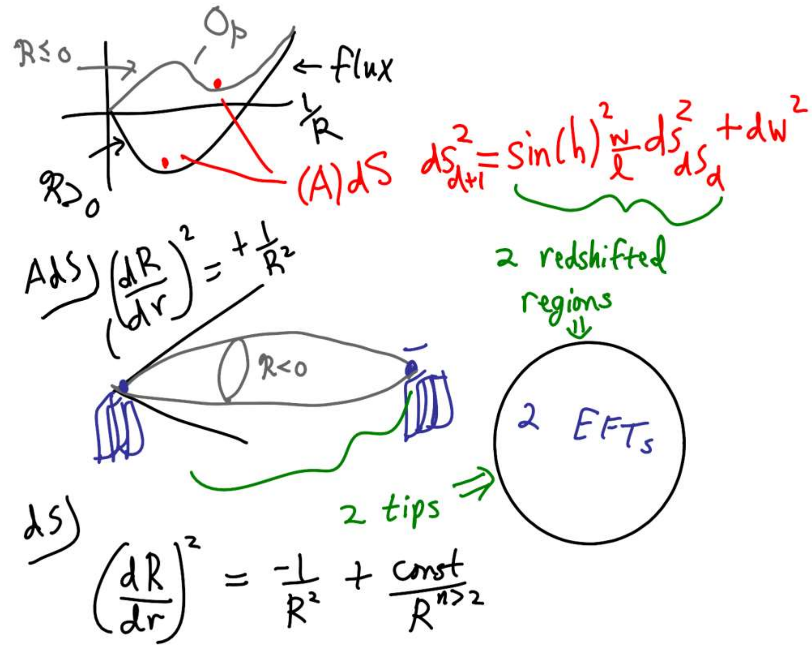
AdS/dS



dS/dS (each point is (d-1)-sphere)

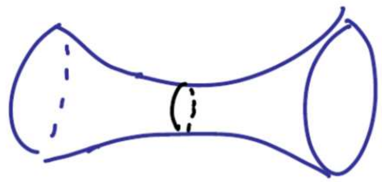
Uplifting AdS/CFT => 2 sectors

Dong Horn ES Torroba ‘10



dS vs AdS brane construction:
independent derivation of the two sectors because of metastability.
Also true in dS/CFT

General idea: entanglement and other properties of the quantum state are tied to the knitting together of spacetime.



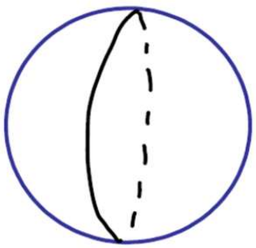
Thermal state of two CFTs (entangled at thermal scale) dual to joined spacetime

$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |n\rangle |n\rangle \,.$$

``ER=EPR"

Van Raamsdonk, Maldacena/Susskind

dS case: Entangled state of 2 deformed CFTs, dominated at the most UV scale ($\Delta \simeq c/6$). Strong interactions between them suggested that this state could be highly mixed.

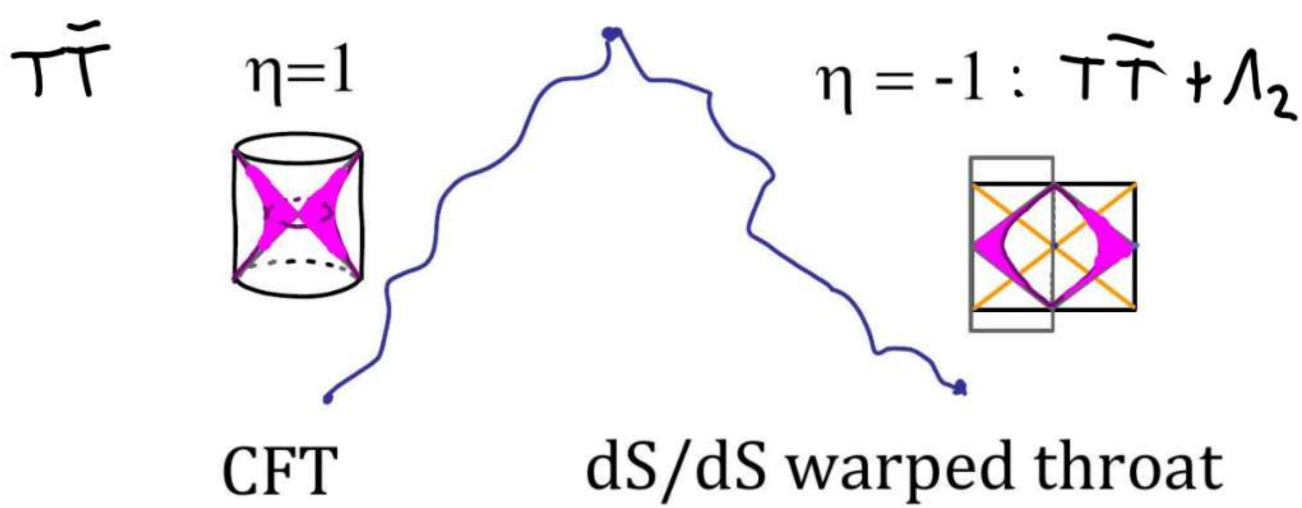


Dong ES Torroba '18, Gorbenko ES Torroba '19, Shyam '21, ...

dS/dS warped throat from the
$T\bar{T} + \Lambda_2$ deformation (GST)

$$
\begin{aligned}
c &= \frac{3\ell}{2G} \\
\lambda &= 8G\ell \\
r &= \ell \sin\left(\frac{w_c}{\ell}\right) \\
L &= 2\pi\mu\ell \sin\left(\frac{w_c}{\ell}\right) \\
\Rightarrow \quad \Lambda_2 = -\frac{1}{\pi\lambda} &= -\frac{c}{12\pi}\frac{1}{r^2} \qquad (w_c = \ell\pi/2)
\end{aligned}
$$

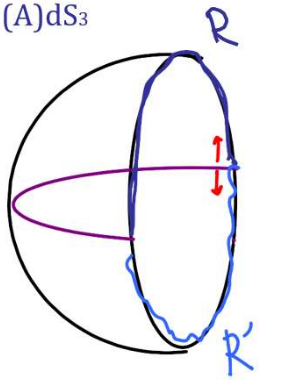
$c\lambda/r^2 >> 1$



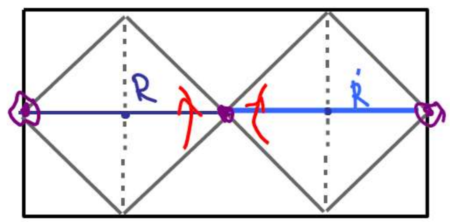
We can analyze the mixing in 2-3 ways: (i) split each throat, then join to obtain full gravity as the last step (a) $T\bar{T} + \Lambda$ analysis (b) gravity path integral or (ii) divide full neck (gravity present from start). Flat spectrum, all.

Rotated calculation: Consider first ½ of a dS/dS warped throat: ¼ of the dS neck

spatial slice of bulk (A)dS_3



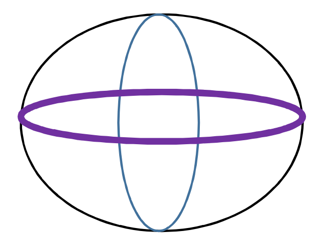
Boundary dS_2



($T\bar{T}$ method: Calculation of Renyis via dressed stress energy => max mixing: LLST)

$$S_0(r) = S_1(r) = \frac{\pi c}{6}$$

Here we have a frozen boundary (D wall), so can calculate as as in Lewkowyzc/Maldacena Dong. Replicate Euclidean throat (w/boundary $EdS_2 = S^2$), getting surfeit angles which smooth out in bulk. Orbifold, giving a cosmic brane which sources deficit angles in bulk.



Now join two such throats and integrate over the shared metric to recover the global dS neck (incorporating the dynamical gravity of the joined system). The integral over the shared metric yields a simple saddle containing the fully back reacted **cosmic brane**. The back reaction is the orbifold-induced deficit angle, leaving the area of the cosmic brane independent of n. (Contrast AdS case of fixed area states: there the area integral => n-dependence Dong Harlow Marolf, Akers Rath)

$$|z_1|^2 + |z_2|^2 = \ell_{dS}^2$$

$EdS_{d+1=3}$

$$(z_1, z_2) \simeq (e^{2\pi i/n} z_1, z_2)\,.$$

Cosmic brane = fixed locus

$$z_1 = 0, \qquad |z_2|^2 = \ell_{dS}^2\,.$$

=> $S_n = S_{VN}$ at large c

Original calculation DST:

Divide the global dS neck in 2 (to calculate ρ_A). Replicate, smooth out surfeits (everywhere, since gravity is dynamical). This gives the original EdS_3. Orbifold => same cosmic brane (just rotated by dS symmetry).

$$|z_1|^2 + |z_2|^2 = \ell_{dS}^2$$

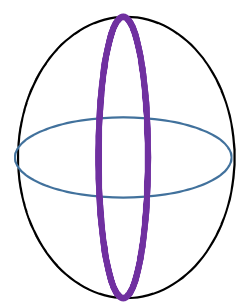
$EdS_{d+1=3}$

$$(z_1, z_2) \simeq (e^{2\pi i/n} z_1, z_2)\,.$$

Cosmic brane = fixed locus

$$z_1 = 0, \qquad |z_2|^2 = \ell_{dS}^2\,.$$

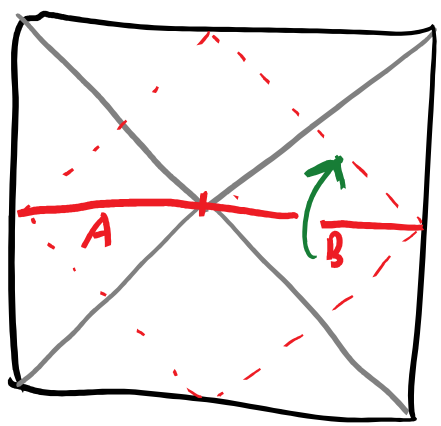
=> $S_n = S_{VN}$ at large c

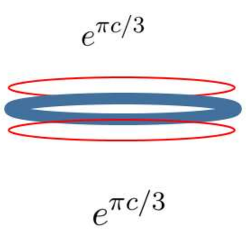


The static patch Hamiltonian is the Modular Hamiltonian K for dS/dS

$$K = -\log(\rho_A)$$

with ρ the reduced density matrix for 1 of the 2 dS/dS warped throats





$$S_{GH} = -Tr\rho_A \log(\rho_A) = S_{EE} = \log(\dim H_{T\bar{T}+\Lambda_2})$$

Dong ES Torroba '18 Coleman Mazenc ES Shyam Soni Torroba Yang '21